

Every finite complex is the classifying space for
proper bundles of a virtual Poincaré duality group.*

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Abstract

We prove that every finite connected simplicial complex is homotopy equivalent to the quotient of a contractible manifold by proper actions of a virtually torsion-free group. As a corollary, we obtain that every finite connected simplicial complex is homotopy equivalent to the classifying space for proper bundles of some virtual Poincaré duality group.

1 Introduction

Let G be a discrete group. A G -CW-complex is, by definition, a CW-complex on which G acts by permuting the cells and cell stabilizers act trivially on cells. A G -CW-complex Y is said to be a model for $\underline{E}G$ if every cell stabilizer is finite and, for every finite subgroup $H \leq G$, the fixed set Y^H is contractible. The existence of such a model can be established by Milnor's or Segal's argument for the construction of the universal space for G . (See [12] for the general construction). Applying an equivariant obstruction theory proves that any two models for $\underline{E}G$ are G -homotopy equivalent. We call $\underline{E}G$ the classifying space for proper G -actions. We write the quotient $\underline{E}G/G$ by $\underline{B}G$ and call it the classifying space for proper G -bundles. Our main theorem can be stated as follows.

Theorem 1 *For any finite connected simplicial complex X , there exists a virtually torsion-free group G with $\underline{E}G$ a cocompact manifold such that $\underline{B}G$ is homotopy equivalent to X .*

A group Γ is called a *Poincaré duality group of dimension n* if $H^i(\Gamma; A) \cong H_{n-i}(\Gamma; A)$ for any $\mathbb{Z}\Gamma$ -module A . Many interesting examples of Poincaré duality groups are manifold groups. More specifically, the fundamental group of a closed, aspherical n -dimensional manifold is a Poincaré duality group of dimension n . (The converse is false.) See [2], [4] for details about Poincaré duality groups. Finally, recall that a group *virtually* has some property if a finite index subgroup has the property.

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Let T be a torsion-free finite index subgroup of G in Theorem 1. Then one can take $\underline{E}G$ as a model for $\underline{E}T = ET$, where ET is the universal space for T . Since ET/T is a closed aspherical manifold, T is a Poincaré duality group.

Corollary 2 *For any finite connected simplicial complex X , there is a virtual Poincaré duality group G such that $\underline{B}G$ is homotopy equivalent to X .*

The statement of Corollary 2 is related to the theorem of Kan-Thurston, which says that every connected complex has the same homology as the classifying space for some group (See [9]). This theorem has been extended and generalized by a number of authors. For example, see [1],[13],[8],[11],[14],[12],[10]. Among many extensions and generalizations, Leary and Nucinkis proved in [12] that every connected CW-complex has the same homotopy type as the classifying space for proper bundles of some group. Corollary 2 says that the group can be taken as a virtual Poincaré duality group if the simplicial complex is finite.

The proof of Theorem 1 consists of three steps. In Section 2, we outline the embedding trick, due to Floyd [7], for equivariantly embedding a simplicial complex with an involution into some Euclidean space. In Section 3, we review the equivariant reflection group trick, due to Davis [6]. Finally, in Section 4, we use the two tricks to construct a contractible manifold, whose quotient by some group G is homotopy equivalent to a given finite simplicial complex X . We also prove that the contractible manifold is the classifying space for proper G -actions and introduce the torsion-free subgroup of finite index to complete the proof of Theorem 1.

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2 The Equivariant Embedding Trick

Let Z be a finite simplicial complex with a simplicial map of period p . In [7], Floyd introduced the embedding trick, namely, Z can be embedded in some Euclidean space such that the restriction of specific coordinate changing map on Z is the given simplicial map. We outline his construction in the case that $p = 2$. See [7, Section 2] for the full construction.

Let L be a finite connected simplicial complex with a simplicial involution T . Embed L into \mathbb{R}^n for some n and suppose that \mathbb{R}^n is triangulated so that L is a subcomplex. Consider the following map.

$$\phi : L \rightarrow \mathbb{R}^n \times \mathbb{R}^n (= \mathbb{R}^{2n}), \quad x \mapsto (x, T(x)).$$

Note that a cell in the cellular decomposition of \mathbb{R}^{2n} has the type of $s_1 \times s_2$, where each s_i is a simplex in \mathbb{R}^n . We use the first barycentric subdivision of this cellular decomposition for the subdivision of \mathbb{R}^{2n} .

By passing to the barycentric subdivision $\text{Sd}(L)$ of L , we obtain that ϕ is a simplicial homeomorphism of $\text{Sd}(L)$ onto a subcomplex of \mathbb{R}^{2n} . Furthermore, the

map $S : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^n \times \mathbb{R}^n$ defined by $S(x, y) = (y, x)$ is simplicial and satisfies $S \circ \phi = \phi \circ T$, hence ϕ is an equivalence between $(\text{Sd}(L), T)$ and $(\phi(\text{Sd}(L)), S)$. Hereafter, we suppose that L is a subcomplex of \mathbb{R}^{2n} and $S = T$ on L .

We may as well assume that L is a full subcomplex of \mathbb{R}^{2n} . For if not, $\text{Sd}(L)$ is a full subcomplex of $\text{Sd}(\mathbb{R}^{2n})$. Let U be the first regular neighborhood of L , i.e. the union of all open stars of vertices of $\text{Sd}(L)$ relative to $\text{Sd}(\mathbb{R}^{2n})$. Then $K = \overline{U}$ is a manifold with boundary of dimension $2n$. Denote the boundary of K by ∂K .

Let $v_0, \dots, v_k, v_{k+1}, \dots$ be vertices of \mathbb{R}^{2n} , where v_0, \dots, v_k are vertices of L . Every point x in \mathbb{R}^{2n} has a unique barycentric representation $\sum t_i v_i$. Furthermore, K consists of points x for which $\max(t_0, \dots, t_k) \geq \max(t_{k+1}, \dots)$ and ∂K consists of points x for which $\max(t_0, \dots, t_k) = \max(t_{k+1}, \dots)$. Consider $f : K \rightarrow L$ defined by $f(x) = f(\sum t_i v_i) = \frac{\sum_{i=0}^k t_i v_i}{\sum_{i=0}^k t_i}$. Then $\Phi : K \times I \rightarrow K$ defined by $\Phi(x, t) = (1 - t)x + tf(x)$ is a deformation retract of K onto L .

Remark 3 By the unique barycentric representation of points in \mathbb{R}^{2n} , $\Phi(S(x), t) = S(\Phi(x), t)$ for any $x \in K$ and $t \in I$ so that Φ is S -equivariant. Therefore, K equivariantly deformation retracts onto L . In particular, the fixed set K^S is homotopy equivalent to the fixed set L^T and $K/\langle S \rangle$ is homotopy equivalent to $L/\langle T \rangle$.

3 The Equivariant Reflection Group Trick

Suppose that we are given a space M and a subspace $N \subset M$ such that N is triangulated as a finite dimensional flag complex. Recall that a simplicial complex is a *flag complex* if any finite set of vertices, which are pairwise connected by edges, spans a simplex. Let a discrete group G act on M so that G stabilizes the subspace N and G acts on N by simplicial automorphisms. Following [6], we will associate a right-angled Coxeter group W and construct a $(W \rtimes G)$ -action on a space $\mathcal{U}(M, N, G)$.

Let I be a vertex set of N . Define a right-angled Coxeter group W as follows. There is one generator s_i for each $i \in I$. Relations are given by $s_i^2 = 1$ and $(s_i s_j)^2 = 1$ if $\{i, j\}$ spans an edge in N . For $x \in N$, let $\sigma(x) = \{i \in I \mid x \in N_i\}$, where N_i is the closed star of the vertex i in the barycentric subdivision of N and W_x be the subgroup generated by $\{s_i \mid i \in \sigma(x)\}$.

Note that G acts on N by permuting vertices, so we can form $W \rtimes G$.

Define the space $\mathcal{U}(M, N, G)$ by

$$\mathcal{U}(M, N, G) = W \times M / \sim,$$

where $(w, x) \sim (w', x')$ if $x = x'$ and $w^{-1}w' \in W_x$. For $[w, x] \in \mathcal{U}(M, N, G)$ and $(v, g) \in W \rtimes G$, the action of $W \rtimes G$ on $\mathcal{U}(M, N, G)$ is defined by

$$(v, g).[w, x] = [vw^g, g.x].$$

Remark 4 This construction enjoys the following properties. For details or proofs, see [5, Section 11.7], [3].

1. If M is contractible, then so is $\mathcal{U}(M, N, G)$.
2. If M is an n -dimensional manifold with boundary and $N = \partial M$, then $\mathcal{U}(M, N, G)$ is an n -dimensional manifold.
3. Let $C(N)$ be the cone on N . Then $\mathcal{U}(C(N), N, G)$ has a natural CAT(0) cubical structure so that the link of each vertex is isomorphic to N , and so that $W \rtimes G$ acts by a group of isometries. In particular, for any finite subgroup F of $W \rtimes G$, the fixed point set $\mathcal{U}(C(N), N, G)^F$ is contractible.

A group action on a simplicial complex is said to be *admissible* if, for any simplex, setwise stabilizers are equal to pointwise stabilizers. If the G -action on N is admissible, we have the following.

Lemma 5 *Let H be a finite subgroup in G . Then*

$$\mathcal{U}(M, N, G)^H = \mathcal{U}(M^H, N^H, V_H),$$

where M^H , and N^H respectively, is the H -fixed set in M , and N respectively, and $V_H = N_G(H)/H$.

Remark 6 The above lemma is stated in [5, Proposition 11.7.1] without a proof. We provide the proof below. Note also that N^H is a flag complex.

Proof. It is obvious that $\mathcal{U}(M^H, N^H, V_H)$ is a subspace of $\mathcal{U}(M, N, G)$.

Let $[w, m] \in \mathcal{U}(M, N, G)^H$ be given. In order to prove that $\mathcal{U}(M, N, G)^H$ is contained in $\mathcal{U}(M^H, N^H, V_H)$, it suffices to show that $m \in M^H$ and $w \in W_H$, where W_H is the subgroup of W generated by $\{s_i | i \text{ is a vertex in } N^H\}$. For any $h \in H$,

$$\begin{aligned} (1, h) \cdot [w, m] &= [w^h, h \cdot m] = [w, m] \\ \Rightarrow \quad h \cdot m &= m, \quad w^{-1}w^h \in W_m. \end{aligned}$$

Since $h \cdot m = m$ for all $h \in H$, it follows that $m \in M^H$. We prove that $w \in W_H$ by induction on the length $l(w)$ of w . To begin with, we point out that since the G -action on M is admissible, every generator in W_m is fixed by H . In particular, W_m is a subgroup in W_H . Also note that W_m is finite. (Consider the simplex containing m of minimal dimension.)

Suppose that $l(w) = 1$, i.e. $w = w^{-1}$. Since W_m is finite, ww^h has finite order. But this happens only if two vertices corresponding to w and w^h are connected. Admissibility implies that w is fixed by h , and hence, $w \in W_H$.

Suppose that $w = s_1 \cdots s_n$ is a reduced word (of length n). Let $t_i = s_i^h$.

Suppose $s_1 \neq t_1$. Again, $w^{-1}w^h$ has finite order.

$$\begin{aligned} (w^{-1}w^h)^n &= 1 \text{ for some } n \\ \Rightarrow (s_n \cdots s_1 \cdot t_1 \cdots t_n)(s_n \cdots s_1 \cdot t_1 \cdots t_n) \cdots (s_n \cdots s_1 \cdot t_1 \cdots t_n) &= 1 \end{aligned}$$

In order for the left hand side to be reduced to the identity, in particular, there exists t_i for $2 \leq i \leq n$ such that t_i commutes with t_1 and cancels with s_1 , i.e.

$t_i = s_1$. But this is impossible, because s_1 and t_1 do not commute. Therefore, $s_1 = t_1$.

$$w^{-1}w^h = s_n \cdots s_1 \cdot t_1 \cdots t_n = s_n \cdots s_2 \cdot t_2 \cdots t_n \in W_m$$

By the induction hypothesis, $s_1 w \in W_H$, so $w \in W_H$. ■

4 The proof of Theorem 1

The proof of Theorem 1 consists of three steps. First, we use the equivariant embedding trick to embed a given finite simplicial complex X into the manifold M with an involution τ such that $M/\langle\tau\rangle$ is homotopy equivalent to X . Then we apply the equivariant reflection group trick on M with boundary to obtain a contractible manifold on which some group G acts. The quotient of the contractible manifold by G will be homotopy equivalent to X . Finally, we show that the contractible manifold is the classifying space for proper G -actions. Additionally, we introduce a finite index torsion-free subgroup of G , which proves that G is a virtual Poincaré duality group.

Let X be a finite connected simplicial complex. Note that the equivariant embedding trick requires a simplicial complex with a periodic simplicial map. In this paper, we use the construction appearing in [11]. Applying [11, Theorem A], we obtain a finite connected locally CAT(0) cubical complex Y with a cubical involution τ such that $Y/\langle\tau\rangle$ is homotopy equivalent to X . By passing to the barycentric subdivision, we may assume that Y is a finite connected simplicial complex and τ is a simplicial involution on Y . Now we apply the equivariant embedding trick introduced in Section 2 to obtain a manifold M with a boundary N and a simplicial involution ω on M such that M equivariantly deformation retracts onto Y . By passing to the barycentric subdivision, we can assume that N is a flag complex and a cyclic group of order two, $C_2 = \langle\omega\rangle$, acts on N admissibly. Note that Y is locally CAT(0). Therefore, M is aspherical and M^ω is homotopy equivalent to Y^τ .

Next we apply the equivariant reflection group trick from Section 3 to obtain a space $\mathcal{U}(M, N, C_2)$ on which $W \rtimes C_2$ acts, where W is a right-angled Coxeter group associated to N . Let \widetilde{M} be the universal cover of M , \widetilde{N} be the inverse image of N in \widetilde{M} and \widetilde{W} be the associated right-angled Coxeter group to \widetilde{N} . Repeat the equivariant reflection group trick to obtain a space $\mathcal{U}(\widetilde{M}, \widetilde{N}, \Gamma)$ on which $\widetilde{W} \rtimes \Gamma$ acts, where Γ is the group of liftings of the C_2 -action on M to \widetilde{M} . Note that every torsion element in Γ has order at most two and every finite subgroup of Γ is cyclic of order two.

Proposition 7 *Let $\mathcal{U}(M, N, C_2)$ and $\mathcal{U}(\widetilde{M}, \widetilde{N}, \Gamma)$ be the spaces constructed above. Then*

1. $\mathcal{U}(\widetilde{M}, \widetilde{N}, \Gamma)$ and $\mathcal{U}(M, N, C_2)$ are manifolds.
2. $\mathcal{U}(\widetilde{M}, \widetilde{N}, \Gamma)$ is the universal cover of $\mathcal{U}(M, N, C_2)$.

3. $\mathcal{U}(\widetilde{M}, \widetilde{N}, \Gamma)/(\widetilde{W} \rtimes \Gamma)$ is homotopy equivalent to X .

Proof. The first statement follows from the fact that M and \widetilde{M} are manifolds with boundary. It is clear that $\mathcal{U}(\widetilde{M}, \widetilde{N}, \Gamma)$ is a cover of $\mathcal{U}(M, N, C_2)$. Since M is aspherical, \widetilde{M} is contractible and so is $\mathcal{U}(\widetilde{M}, \widetilde{N}, \Gamma)$. This proves the second statement. By construction,

$$\mathcal{U}(\widetilde{M}, \widetilde{N}, \Gamma)/(\widetilde{W} \rtimes \Gamma) \simeq \mathcal{U}(M, N, C_2)/(W \rtimes C_2) \simeq M/C_2 \simeq X,$$

where \simeq is a homotopy equivalence. ■

We compare the $(\widetilde{W} \rtimes \Gamma)$ -action on the space $\mathcal{U}(\widetilde{M}, \widetilde{N}, \Gamma)$ with the same action on $\mathcal{U}(C(\widetilde{N}), \widetilde{N}, \Gamma)$, and prove $\mathcal{U}(\widetilde{M}, \widetilde{N}, \Gamma) = \underline{\mathcal{E}}(\widetilde{W} \rtimes \Gamma)$. Denote the image of $\{\widetilde{w}\} \times \widetilde{M}$ in $\mathcal{U}(\widetilde{M}, \widetilde{N}, \Gamma)$ by $\widetilde{w}\widetilde{M}$ and the image of $\{\widetilde{w}\} \times (\widetilde{M} \setminus \widetilde{N})$ by $\text{int}(\widetilde{w}\widetilde{M})$. First, we prove that all stabilizers are finite.

Proposition 8 *Let H be a subgroup of $\widetilde{W} \rtimes \Gamma$ fixing some point in $\mathcal{U}(\widetilde{M}, \widetilde{N}, \Gamma)$. Then H is finite.*

Proof. Suppose that H fixes some point in $\text{int}(\widetilde{w}\widetilde{M})$ for some \widetilde{w} .

Then $H' = (\widetilde{w}, 1)^{-1}H(\widetilde{w}, 1)$ fixes some point in $\text{int}(\widetilde{M})$. Denote this point by $[1, x]$. For any $(\widetilde{v}, \gamma) \in H'$,

$$(\widetilde{v}, \gamma).[1, x] = [\widetilde{v}, \gamma.x] = [1, x] \Rightarrow \gamma.x = x, \widetilde{v} \in \widetilde{W}_x$$

Since $x \in \widetilde{M} \setminus \widetilde{N}$ (recall that $\widetilde{N} = \partial\widetilde{M}$, and $x \in \text{int}(\widetilde{M})$), \widetilde{W}_x is trivial. It follows that H' is a subgroup of Γ . Recall that Γ is the group of liftings of the C_2 -action on M to \widetilde{M} . Therefore, if a nontrivial element γ fixes some point x , γ is the only nontrivial element in Γ fixing x . This tells us that H' must be finite of order 2.

Suppose that the fixed point is not contained in $\text{int}(\widetilde{w}\widetilde{M})$ for any \widetilde{w} . As in the previous case, choose some \widetilde{w}' so that $H'' = (\widetilde{w}', 1)^{-1}H(\widetilde{w}', 1)$ fixes some point in the image of \widetilde{N} in $\mathcal{U}(\widetilde{M}, \widetilde{N}, \Gamma)$. Denote such a point by $[1, y]$. For any $(\widetilde{v}', \gamma') \in H''$,

$$(\widetilde{v}', \gamma').[1, y] = [\widetilde{v}', \gamma'.y] = [1, y] \Rightarrow \gamma'.y = y, \widetilde{v}' \in \widetilde{W}_y$$

As before, we have at most two possibilities for γ' . Furthermore, \widetilde{W}_y is finite. (Consider the simplex containing y of minimal dimension.) Therefore, H'' is finite, and hence, H is finite. ■

Theorem 9 $\mathcal{U}(\widetilde{M}, \widetilde{N}, \Gamma) = \underline{\mathcal{E}}(\widetilde{W} \rtimes \Gamma)$.

Proof. It suffices to prove that the fixed point set by a finite subgroup is contractible. As mentioned before, we consider the $(\widetilde{W} \rtimes \Gamma)$ -action on $\mathcal{U}(C(\widetilde{N}), \widetilde{N}, \Gamma)$.

Let F be a finite subgroup of $\widetilde{W} \rtimes \Gamma$. Recall that $\mathcal{U}(C(\widetilde{N}), \widetilde{N}, \Gamma)^F$ is contractible. (See Remark 4.) In particular, it is nonempty.

Suppose that F does not fix any cone point in $\mathcal{U}(C(\tilde{N}), \tilde{N}, \Gamma)$. Then F fixes no point in $\text{int}(\tilde{w}\tilde{M})$ for any \tilde{w} . In other words,

$$\mathcal{U}(\tilde{M}, \tilde{N}, \Gamma)^F \subset \mathcal{U}(\tilde{M}, \tilde{N}, \Gamma) - \bigcup_{\tilde{w} \in \tilde{W}} \text{int}(\tilde{w}\tilde{M}).$$

Therefore,

$$\mathcal{U}(\tilde{M}, \tilde{N}, \Gamma)^F = \mathcal{U}(C(\tilde{N}), \tilde{N}, \Gamma)^F$$

and $\mathcal{U}(\tilde{M}, \tilde{N}, \Gamma)^F$ is contractible.

Suppose that F fixes some cone point in $\mathcal{U}(C(\tilde{N}), \tilde{N}, \Gamma)$. Choose some \tilde{w}'' so that $F' = (\tilde{w}'', 1)^{-1}F(\tilde{w}'', 1)$ fixes the cone point of $\text{int}(C\tilde{N})$. Denote the cone point by c . For any $(\tilde{v}'', \gamma'') \in F'$,

$$(\tilde{v}'', \gamma''), [1, c] = [\tilde{v}'', \gamma''].c = [1, c] \Rightarrow \gamma''.c = c, \tilde{v}'' \in \tilde{W}_c.$$

Since c is a cone point, \tilde{v}'' is trivial. It follows that F' is a finite subgroup of Γ , and hence, cyclic of order 2. By Lemma 5, it follows that $\mathcal{U}(\tilde{M}, \tilde{N}, \Gamma)^{F'}$ is $\mathcal{U}(\tilde{M}^{F'}, \tilde{N}^{F'}, V_{F'})$. Recall M is equivariantly homotopy equivalent to a locally CAT(0) space Y . Therefore, $\tilde{M}^{F'}$ is homotopy equivalent to the fixed set of CAT(0) space by a cyclic group of order 2. It follows that $\tilde{M}^{F'}$ is contractible, hence so is $\mathcal{U}(\tilde{M}^{F'}, \tilde{N}^{F'}, V_{F'})$. Finally, $\mathcal{U}(\tilde{M}, \tilde{N}, \Gamma)^F = (\tilde{w}'', 1)\mathcal{U}(\tilde{M}, \tilde{N}, \Gamma)^{F'}$ is contractible. ■

Remark 10 Consider the commutator subgroup T of \tilde{W} and its inverse image \tilde{T} in \tilde{W} . Since \tilde{T} is torsion-free and of finite index in \tilde{W} , $\tilde{T} \rtimes \pi_1(M)$ is a torsion-free finite index subgroup of $\tilde{W} \rtimes \Gamma$. Since $\mathcal{U}(\tilde{M}, \tilde{N}, \Gamma)/(\tilde{T} \rtimes \pi_1(M))$ is an aspherical closed manifold, $\tilde{T} \rtimes \pi_1(M)$ is a Poincaré duality group. This verifies that $\tilde{W} \rtimes \Gamma$ is a virtual Poincaré duality group.

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